

Chapter 3: Basic wave concepts

In this chapter, I will introduce ideas pertaining to waves. I will give examples of waves, discuss ways of classifying them, and describe some of the things waves can do.

Types of waves

Definition and examples

Waves are physical disturbances that transmit energy. Often, a *medium* of transmission is involved, but not always. One example of a medium is the air that most sounds we hear travel through. Not all waves require media; light, for example, can travel through empty space. Still, light transmits energy. Even with the presence of a medium, a wave can travel a very long distance with the individual pieces of medium only moving a short distance. We'll see this fact in the videos linked below.

I typically start a class on waves by soliciting examples. As a reader, you can think of some now!

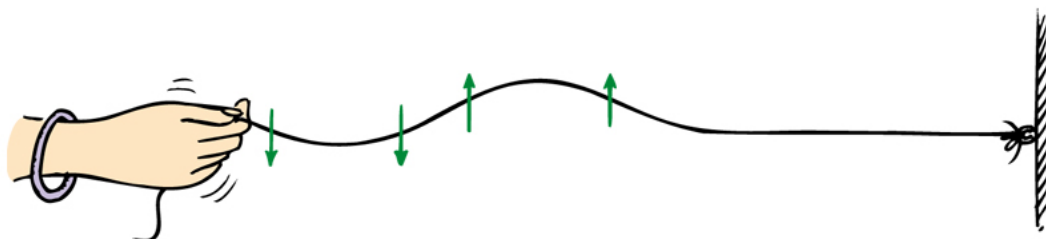
Often, examples that come up include sound, light, water waves, microwaves, and radio waves. Three of these (light, microwaves, and radio) are examples of *electromagnetic waves*, and they all share certain physical properties. Occasionally other examples come up in discussion: seismic waves (related to earthquakes), “the wave” as done at a sports stadium, water waves, and X-rays are other popular examples.

Classification

Though I mentioned just now that certain types of waves are classified as *electromagnetic* due to their origin, velocity, etc., there is a more general way to classify waves. Fortunately, study of a spring or Slinky-like toy can demonstrate both the major classifications.

Waves are either termed *transverse* or *longitudinal* depending on how the medium moves compared to the direction of wave travel. The medium in a *transverse wave* moves perpendicular to the direction the overall disturbance moves. Strings on musical instruments vibrate this way. To see an animation of a transverse wave, watch the video [here](#).

Longitudinal waves feature media that move parallel to the direction of wave propagation. [This video](#) shows a longitudinal pulse being made. Sound waves are the best example of longitudinal waves. If you need help picturing this, think about what a speaker does! The cone pulses “in and out” rather than “up and down.”



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Some transverse wave pulses. The wave moves to the right and the string pieces go up and down.

Both my previous definitions discuss the medium the wave travels through. The astute reader may have noticed that I mentioned how electromagnetic waves do not require a medium to travel through. These types of waves can be classified, though a bit more abstractly. Electromagnetic waves contain intertwined electric and magnetic fields that oscillate in perpendicular directions that are BOTH perpendicular to the direction of wave travel, so EM waves are considered transverse.

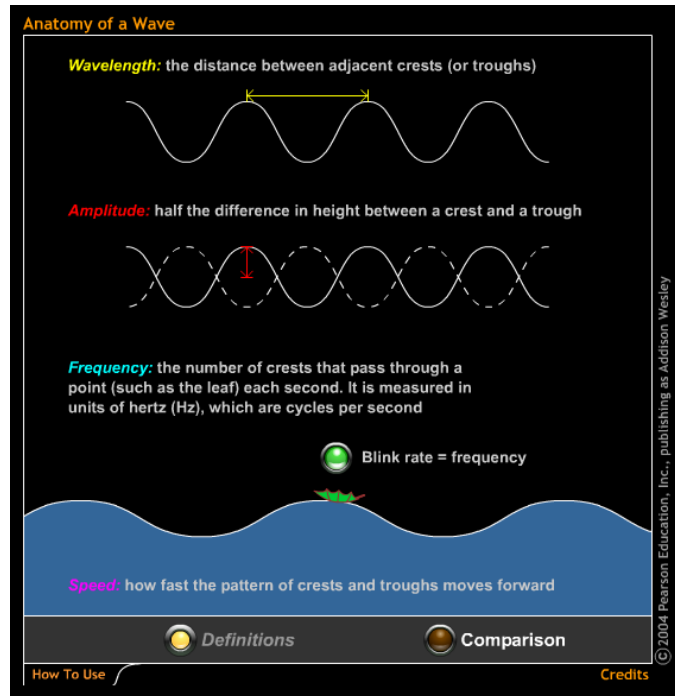
I often like introducing “the wave” at sporting events as a type of wave. It counts, and people are the medium! If you think about it, when doing “the wave,” the disturbance travels left to right or vice versa, and as part of the disturbance, your role as spectator is to stand up. Therefore, your motion (as a part of the medium) is perpendicular to the wave propagation and “the wave” counts as transverse!

The anatomy of waves

Use [this simulation](#) to picture a traveling wave. Water waves are often intuitive to think about, so we’ll start there. In the simulation, some water waves pass by and cause the leaf to move up and down on a regular basis. Suppose, just for the sake of putting in some numbers, that the wave peaks are three meters apart and two wave peaks pass by every second.

Wave speed

We might ask the question: how *fast* is the wave disturbance moving to the right? Well, if the leaf is currently at a peak, and two peaks will arrive in the next second, we can figure the wave must be traveling at 6 m/s. That way, the wave will travel six meters in one second, and that will allow for two peaks to hit the leaf.



The leaf simulation linked above (from Addison Wesley)

It helps to attach some terminology to this problem. “Three meters” represents the *wavelength* of the wave pattern. Wavelength is defined as the distance between adjacent crests or troughs if you were to take a snapshot of the wave in time. “Two peaks arriving per second” can be written as 2 Hz and is therefore a unit of *frequency*. 6 m/s is the wave speed, and you might now notice that the wavelength multiplied by frequency gives you the wave speed! Usually, the Greek letter lambda (λ) is used to signify wavelength. If we use v (short for velocity) to signify wave speed and f for frequency, we can write:

$$v = f\lambda$$

and this relationship holds for all waves! This course will break many parts of this equation down: we can talk about the speed of sound in different conditions as well as how frequency and musical pitch are related.

Example: Let’s do an example using a musical note. Suppose we play a pure tone of a frequency equal to a piano’s middle C (262 Hz) indoors at normal room temperature. The speed of sound in these conditions (assume the temperature is 20°C or 68°F) is 343 m/s. What is the wavelength of the sound waves?

Well, it’s often a good idea to rearrange the equation first so there are fewer places to make calculator errors. If you divide both sides of the wave speed equation by f , you get

$$\frac{v}{f} = \lambda$$

so the wavelength then is $(343 \text{ m/s}) / (262 \text{ Hz})$. Since Hz is another way to write “cycles per second,” we get “meters per cycle” for the units of our answer – which is what a wavelength is! The final answer is that the wavelength is 1.31 m (about 51.6 inches). That means that in the repeating sound wave patterns, locations of maximum air compression are 1.31 m apart.

Light and electromagnetic waves

As mentioned earlier, light is a type of wave¹ that does not require a medium. The light we see every day is a special type of a far more general class of waves: electromagnetic waves. In fact, there exists a large *electromagnetic spectrum* that encompasses all sorts of radiation. I will discuss some of these types as well as where visible light fits in and the properties of these waves.

Speed of electromagnetic waves

All electromagnetic waves in vacuum travel the same speed, commonly called the *speed of light* after the most famous of electromagnetic waves. This speed is considered a “universal speed limit” because it is impossible for anything having mass to reach this speed, and no known waves travel faster. The letter c is used to represent the speed of light (in vacuum) in physics, and it has the value

$$c \approx 3 \times 10^8 \text{ m/s}$$

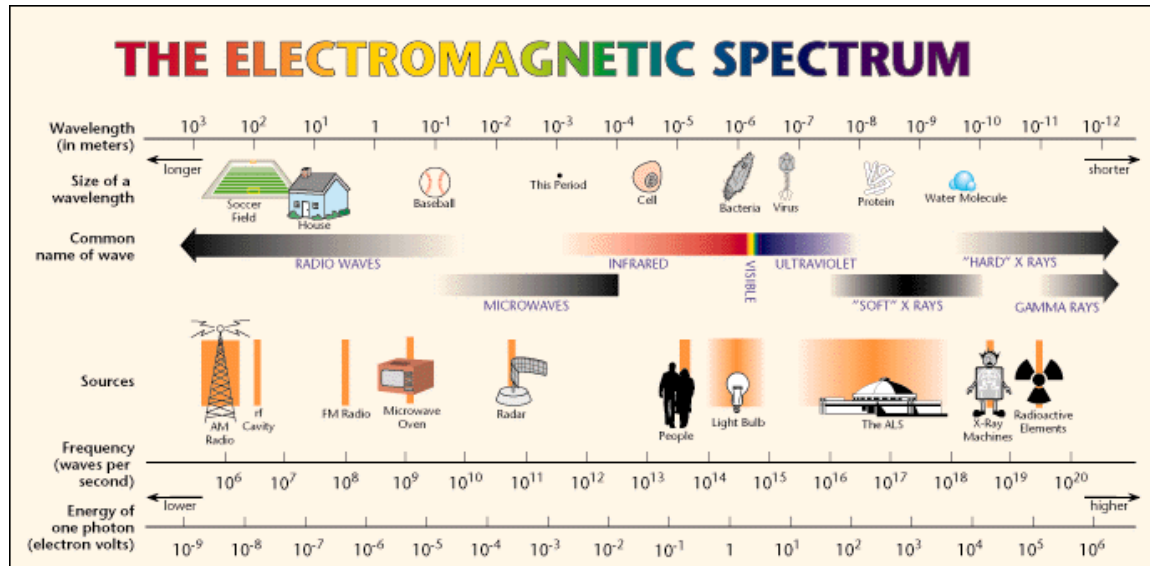
which is about 186,000 miles per second.

Albert Einstein’s special theory of relativity posits that all light in vacuum travels this speed no matter the speed of the source and receiver of the light – that is, if the starship *Enterprise* is going half the speed of light and turns on the headlights, an approaching Klingon ship would see the light traveling toward it at the same speed as when the *Enterprise* is still!

I mentioned what the speed of light is in vacuum and that it is a universal speed limit. However, light and other electromagnetic waves can go slower under certain conditions. Materials like water and glass slow these waves down by a factor equal to their *index of refraction* (a number that goes into calculating how much light bends in, say, a prism or air/water boundary). Most glass slows light down by a factor of 1.5, and water slows light less – by a factor of 1.33.

¹ Yes, light sometimes behaves like particles. Some phenomena can be explained better with this model of light, but we focus on the wave behavior here.

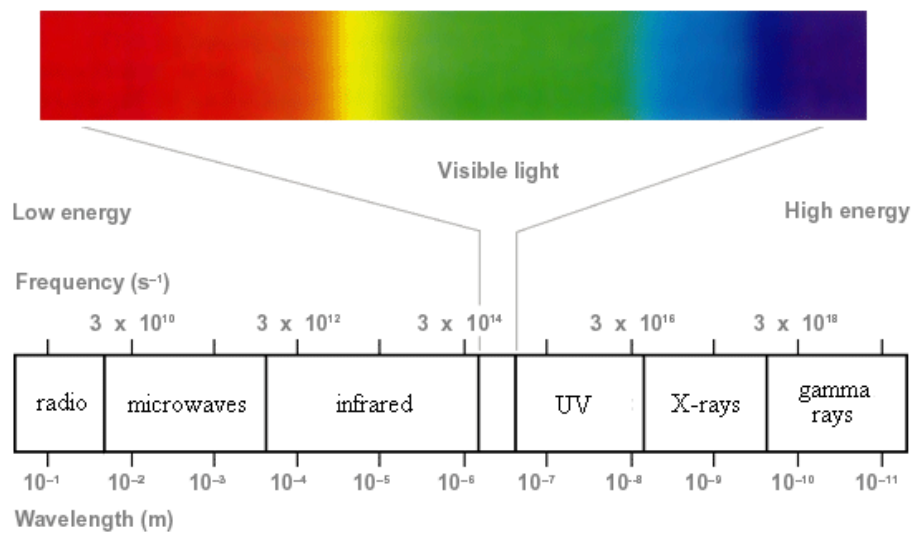
Types of electromagnetic waves and the EM spectrum



The EM spectrum ([source](#))

Visible light is a very small part of the electromagnetic spectrum shown on the last page. Many waves we deal with on a daily basis make up parts of the spectrum: radio waves have the lowest frequency, then as we increase frequency we see microwaves, infrared radiation, visible light, ultraviolet, x-rays, and gamma rays.

The visible light spectrum can be seen wedged between infrared and ultraviolet on the diagram above. Zooming in, we might see something like this:



Visible spectrum (adapted slightly from [here](#))

Red light has the lowest frequency of visible light and violet light has the highest visible frequency. Both colors travel the same speed, though, so the violet light has the smallest wavelength in the visible spectrum (400 nm is the wavelength for violet light, and 700 nm is the wavelength for red). Frequencies lower than that of red light make up *infrared* and frequencies above that of violet light are called *ultraviolet*. We can see on this figure that visible light doesn't even make up one whole tick mark worth of frequencies on this logarithmic scale. There are over 10 powers of 10 represented in the frequencies of EM radiation, and light doesn't even account for a full power of 10.

EM wave math example: Although (as I perceive it) radio is losing popularity to portable music players, we can still do a calculation on a radio wave and determine its wavelength. FM stations in the US refer to a frequency in megahertz (MHz). One local station in Chicago – the NPR station - is 91.5 FM. That means those radio waves have a frequency of 91.5 MHz or 9.15×10^7 Hz. If you want to find the wavelength of these waves, recall that from a previous example

$$\frac{v}{f} = \lambda$$

so

$$\lambda = \frac{3 \times 10^8 \text{ m/s}}{9.15 \times 10^7 \text{ Hz}} = 3.28 \text{ m}$$

which is a little shy of 11 feet.

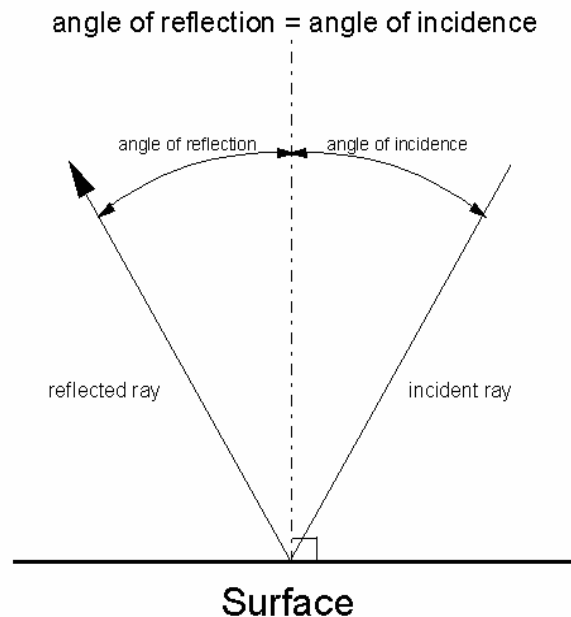
Wave behaviors

Waves interact with matter in a number of ways. They can also interact with each other! Here are some typical wave behaviors.

Reflection

Reflection happens when a wave bounces off the interface between two media. A reflected wave remains in the medium it starts in. The easiest example to “see” is when you look in a mirror; you see an image of yourself because light from the sun or artificial sources scatters off your face, bounces off the silvered back surface of a mirror, and returns to your eyes. Of course, sound reflects off of walls as well, and we'll discuss those reflections in more detail later.

Mirror reflections are called *specular* whereas more scattered reflections (like sound bouncing off a wall) are called *diffuse*. Specular reflections obey the *law of reflection* depicted below. In a sentence, the law of reflection states that the incoming angle of a wave and the outgoing angle of that wave upon reflection are equal.



Specular reflection with equal incident and reflected angles. ([Source](#))

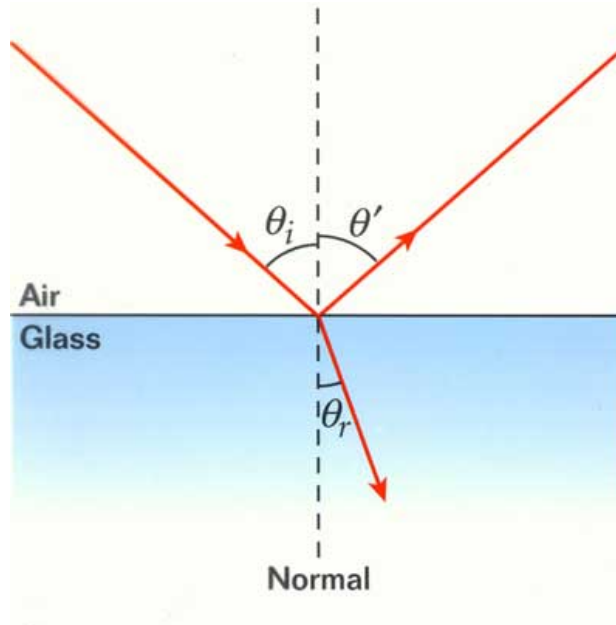
Refraction

Not all waves bounce off an interface. Some (and light is the most well-known perpetrator) bend upon entering another surface, and this process is called *refraction*. You would study this phenomenon more in an optics class because refraction is what allows things like lenses (and the associated technologies of microscopes and telescopes).

All media through which light can travel have an *index of refraction*. Symbolically, this index is represented by the letter “n,” and it represents the factor by which light moves slower in the specific medium compared to vacuum. Empty space, therefore, has an index of refraction of 1, and air has an index very close to 1.

$$n_{\text{medium}} = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}}$$

Water’s index of refraction is about 1.33, and glass (depending on the type) has an index of refraction in the ballpark of 1.5 – hence, glass tends to bend light more than water does. *Snell’s Law* gives the relationship between the index of refraction in materials and the angles of the “incident” and “refracted” beams. I show this law below after the illustration:



θ_i is the incident angle and θ_r is the refracted angle. Light going into glass bends toward the perpendicular imaginary line because glass has a higher refractive index. ([picture source](#))

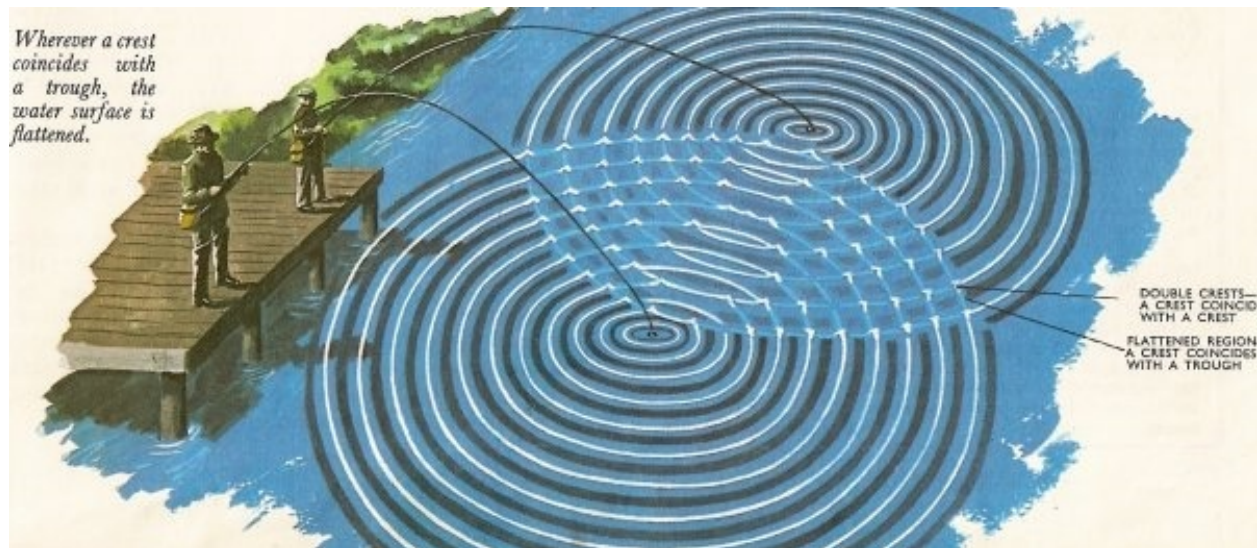
Snell's Law states: $n_i \sin \theta_i = n_r \sin \theta_r$

Interference

Many acoustical phenomena we will discuss later involve the phenomenon of *interference*. Two of these are standing waves and beats. We get interference when waves from two sources meet at one place in space and then interact with one another.

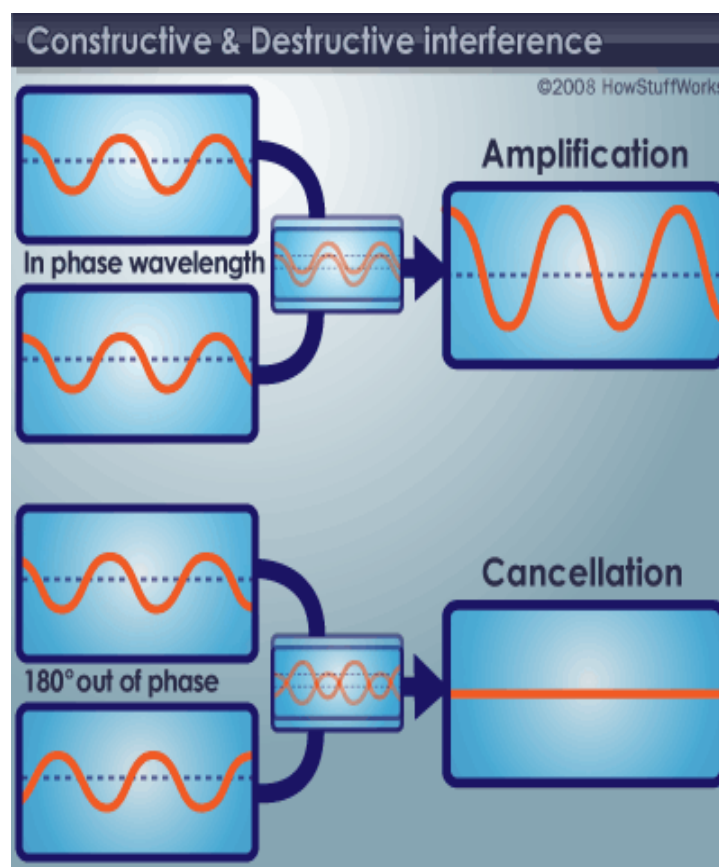
The best way to see this is with a ripple tank. Click [here](#) for a video of an actual tank and [here](#) for a computer animation that is a little clearer. You can see in those videos that there will be some places where the water is still and some where the water vibrates a lot more.

What causes this? Well, the *principle of superposition* tells us what happens when you have two wave sources and their effects interact at a point. This principle says that the net result of two waves interacting at a point is the algebraic sum of what each wave source is doing individually at that point. Therefore, if one spot in the water is at a location where each source would be producing a peak on its own, you'd get a taller peak. If a second spot is at a location where one source would produce a peak and the other source would be producing an equal-sized trough, the net result would be a still spot.



Interference pattern in water waves – still photo

If we look at what happens at each individual spot one at a time, we can characterize what's going on in terms of whether the two sources have effects that “line up” or not – see the picture below:



Interference picture ([source](#))

The diagram shows the two extremes one can have with wave interference. In *constructive interference*, the two sources do the same thing at all times, and the waves add up as much as they possibly can. In the second case, called *destructive interference*, you get cancellation in all instances the two waves cancel out.

You might also notice that the *constructive interference* picture is called “in phase.” The *phase difference* between the two waves is a representation of how much one wave is shifted with respect to the other, and there are a few ways of expressing this. One way is to assign the 360° in a mathematical cycle (or circle) to a full shift in wavelength (λ) between one wave and the next. Two waves that are in phase can also be called “0° out of phase.” The out-of-phase waves are called “180° out of phase.” That number is used because 180 is half of 360, and a shift of half a wavelength between two waves of equal wavelength results in destructive interference. Other phase shifts are possible too – for example, a 90° phase difference involves the two waves shifted by a quarter wavelength (and the result is somewhere between the extremes pictured above).

Real-life interference situations have total constructive interference, destructive interference, and everything in between going on at different places at different times. The next chapter on standing waves shows how a stable wave pattern can form due to interference of multiple waves.

Mathematics of traveling waves

Here is an optional mathematical section about the mathematical functions that describe traveling waves. If any of the expressions using the sine function look unfamiliar, see a lengthier discussion of that math in the appendix posted along with these notes. I reintroduce the key wave variables there as well.

Any function that travels along the x-axis in the positive direction with velocity v can be written in the form

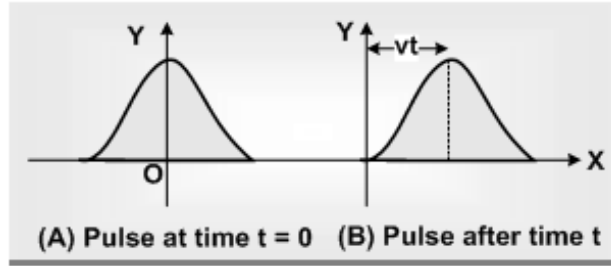
$$y(x, t) = f(x - vt)$$

The way to see that this is a rightward-traveling function is this: at time $t = 0$, suppose you have a wave pulse centered around $x = 0$. The shape of the function is then given by

$$y(x, 0) = f(x)$$

The maximum value of this function is at $x = 0$, and it has the value $f(0)$. At a later time t , the distance the pulse travels must be equal to vt (think of it as a rightward distance equaling speed multiplied by time.) Therefore, the peak of the function is located at $x = vt$. This checks out, because

$$y(vt, t) = f(vt - vt) = f(0)$$



Picture of a traveling wave pulse moving to the right with speed v . ([source](#))

Likewise, a function of the form

$$y(x, t) = f(x + vt)$$

represents a wave traveling to the left with speed v .

Now, for a periodic traveling *sine* wave, we want a function that has the properties discussed above: for a given point in space, motion should repeat with period T , and for a given snapshot in time, the wave pattern should repeat each wavelength λ . The form our wave equation takes will be:

$$y(x, t) = A \sin\left(\frac{2\pi(x - vt)}{\lambda}\right)$$

The A term represents the *amplitude*, or maximum size, of the wave. This looks messy, but we can make sense of it. Notice that I isolated the $(x - vt)$ part to guarantee this would be a valid wave function.

Next, note the 2π factor within the argument of the sine function (the *argument* is all the stuff the sine function acts on, in this case, everything in the large parenthesis). This factor gives us the periodicity we want. The sine function repeats itself if its argument proceeds through 2π radians. The value $(x - vt)$ has a dimension of length, and if this value goes, say, from 0 to λ , the whole argument of the sine function will go from 0 to 2π as we want, and the function repeats. Again, all this makes sense only if radians make sense to you, so check the Appendix if they don't.

This expression for a traveling sine wave does not have all the visually apparent features we want. We can rearrange this and use our $v = f\lambda$ relationship to simplify.

$$y(x, t) = A \sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi vt}{\lambda}\right)$$

$$y(x, t) = A \sin\left(\frac{2\pi x}{\lambda} - 2\pi ft\right)$$

$$y(x, t) = A \sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}\right)$$

Here, going through a wavelength λ in x or a period T in time leads to a 2π change in the argument of the sine function, and the function repeats.

Another common way to write this sine wave function is by introducing the *wave number* (called k) and the *angular frequency*, ω .

$$y(x, t) = A \sin(kx - \omega t)$$

Here, $k = 2\pi / \lambda$ and $\omega = 2\pi / T$. They have units of m^{-1} and s^{-1} , respectively. Sometimes, a phase angle ϕ is added to allow representations of the phase shifts discussed above. The final form then is written:

$$y(x, t) = A \sin(kx - \omega t + \phi)$$

Also, since $\omega = 2\pi f$, we can also write wave speed in terms of ω and k :

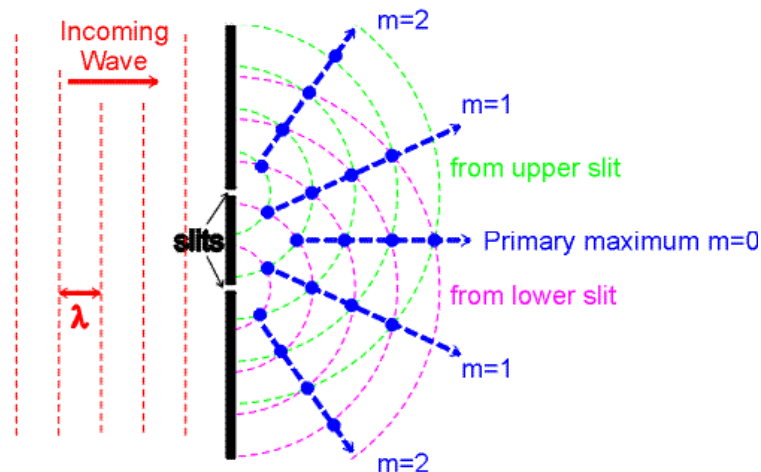
$$v = f\lambda$$

$$v = \left(\frac{\omega}{2\pi}\right) \left(\frac{2\pi}{k}\right) = \omega/k$$

Other examples of interfering waves

My examples above show water waves interfering, and sound waves interfere the same way. A striking visual example comes when looking at interference in light.

When light waves approach a pair of slits, each slit then acts like an individual light source much like the water tank sources made individual ripples (for more detail, look up the [Huygens Principle - video](#)). This is called Young's Experiment. A picture is shown below:

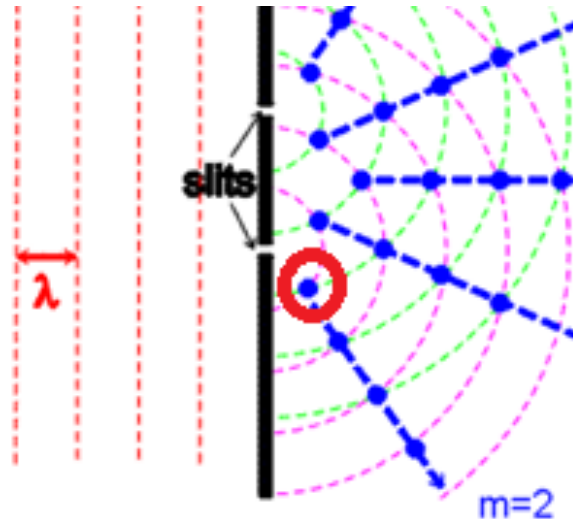


An incoming wave upon two slits. One slit produces waves corresponding to the green dashed lines and the other slit produces the magenta wave fronts. ([source](#))

Typically, this experiment is done with laser light since laser light is *monochromatic* – that is, having one single color and frequency. In the picture above, you can see that there are some spots where peaks

from the two sources meet. There is a pattern! All these places occur along straight lines, and if you put a screen on the right hand side of the setup shown above, you'll get a bright spot where all those lines intersect the screen. You will also get dark spots in between – for at those places, destructive interference is happening.

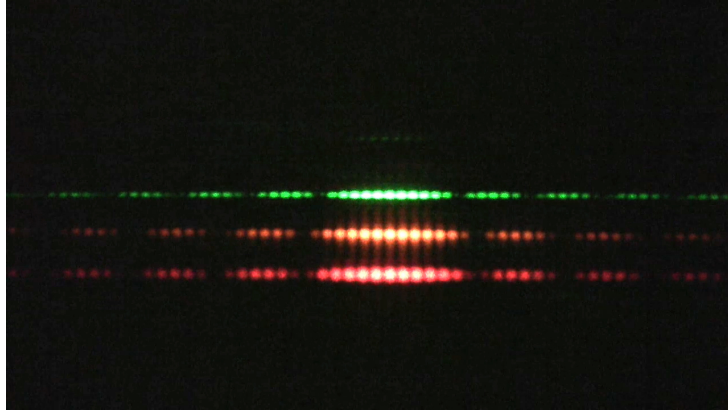
I should clarify what the m in the above figure means because this is a useful idea in other interference problems. Consider in the picture above one of the leftmost points on either line marked " $m = 2$," like the one I've circled below in the zoomed picture.



If you do a quick inspection, you'll see that the point we chose is one magenta wavelength away (λ) from the lower slit and three green wavelengths (3λ) from the upper slit. The two wavelengths are the same, we say the *path difference* is $\lambda \times (3-1)$ or 2λ . That number 2 is the m the diagram is referring to.

So what's going on? In a situation with two wave sources putting out identical wavelengths in phase, you'll get constructive interference anywhere the path difference is an integer number of wavelengths, that is, $m = 0, 1, 2$, etc. If the path difference is some odd number of half wavelengths (like $m = 1/2, 3/2, 5/2$, etc.), a crest from one source will always be meeting a trough from the other, and we'll get destructive interference. [This page](#) shows the $m = 1/2$, etc., lines in the diagram about halfway down.

The pattern shown on a screen for three different colors shining through a double slit is pictured below:



A two-slit interference pattern ([source](#), including video!)

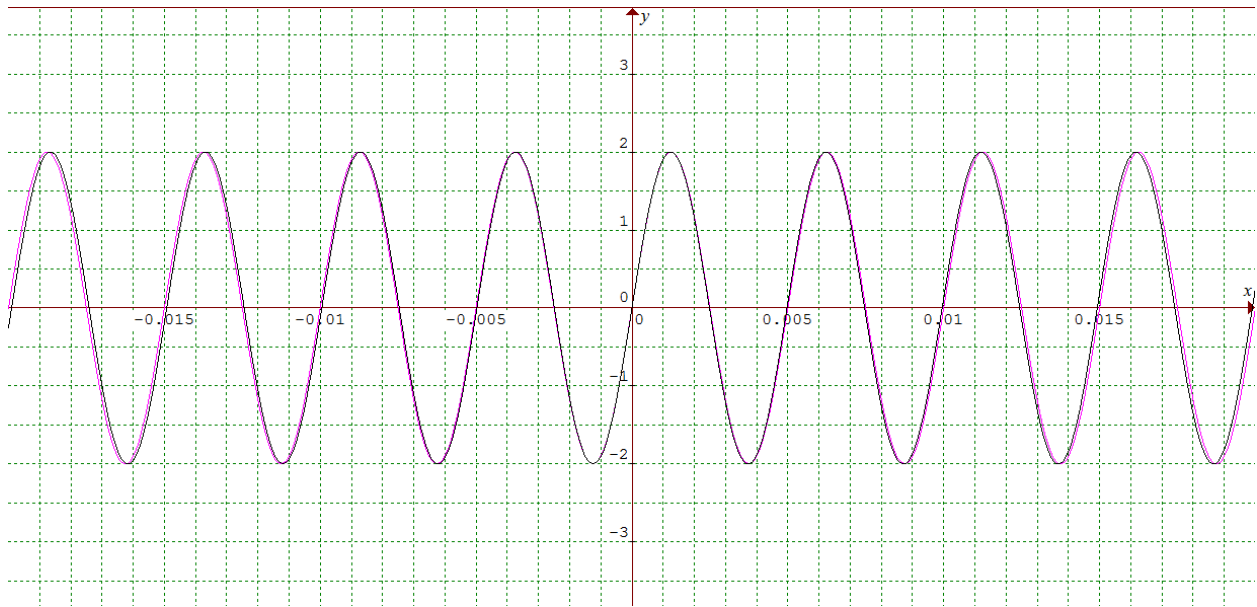
Though we can't see a pattern of ripples interfering like in the water tank video, we can see evidence of the interference on the screen.

Quantum physics diversion: This is only for the interested – it turns out that light and water aren't the only kinds of waves you can see interference with. One can also see interference patterns with things that seem to us like particles (for example, electrons). What's more, you don't need a beam of electrons to see the effect; electrons exhibit the interference pattern even when fired through the slits one at a time. An electron is capable of *self-interference*, and this is possible because even though we think of it as a particle, it exhibits wave behavior. This idea came about in the work of Louis de Broglie in the 1920s. Building off the work of Max Planck and Albert Einstein on the wave-particle "duality" of light, de Broglie extended a hypothesis that electrons have an associated wave whose wavelength depends on the electron's momentum.

The interference patterns you get with electrons (though spaced differently than the laser light) look a lot like the pattern of spots shown above. In fact, scientists have shown interference patterns for increasingly large particles; within the 2000s, even buckyballs (molecules containing 60 carbon atoms) have been shown to exhibit wave-like behavior in the form of interference patterns.

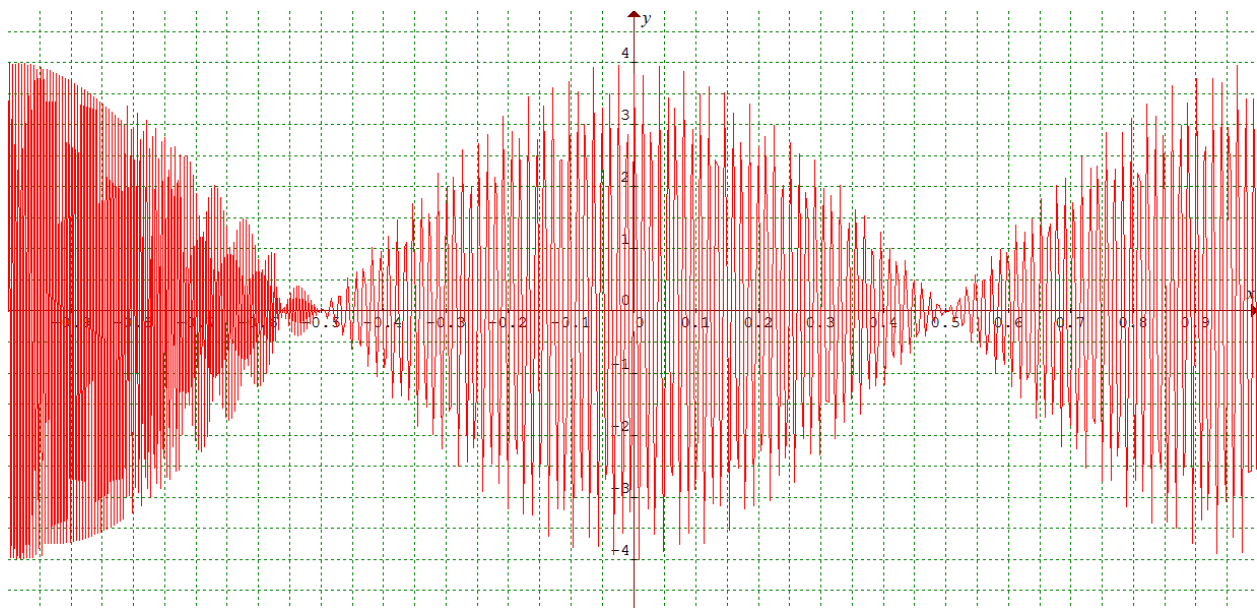
Beats

Moving on from moving sources, let's look at what happens when two similar sound sources are playing at the same time. Suppose one plays a 200 Hz sine-wave tone at the same time as a 201 Hz sine-wave tone. I will graph what these two signals look like together using [Graphmatica](#). (DEMO!)



200 and 201 Hz tones graphed at the same time

These look *nearly* identical; notice on the outer reaches, the black and magenta lines are slightly offset. Also, note that it takes both waves about 0.005 seconds to repeat. 0.005 s is the period that corresponds to a 200 Hz wave. Now, observe what happens when we add these waves together:



The same two waves, added together and zoomed out

The maximum amplitude of the combined wave is 4, just as we'd expect. But now see that the amplitude changes quickly. The sound here is loudest at the zero second mark and again at one second (the far right end of the graph).

The changing amplitude can be heard as a “wobbling” sound or volume swell, and here we see they happen once per second in our example. We say the *beat frequency*, or the frequency of the volume swells that occurs when playing two similar frequencies, is 1 Hz here. In general, the beat frequency is given by:

$$f_{beat} = |f_1 - f_2|$$

where f_1 and f_2 are the two frequencies being played and the absolute value sign gives us a positive answer. Therefore, for example, if sine tones of 440 and 437 Hz are played at the same time, they would produce beats occurring three times every second since $440 - 437 = 3$. The frequency you hear swelling in the beat pattern is the average of the two frequencies you play.

Musically, beats are useful because they can help identify mismatches in tuning between two instruments or two strings on the same instrument. I’ve demonstrated beat phenomena in [this Youtube video](#).

The frequency your ears perceive is the average of the two, i.e.,

$$f = \frac{f_1 + f_2}{2}$$

For a simple example, if two frequencies of 220 Hz and 222 Hz are played (like I did in the video above), the beat frequency will be 2 Hz and the frequency your ears perceive will be 221 Hz.

Homework problems

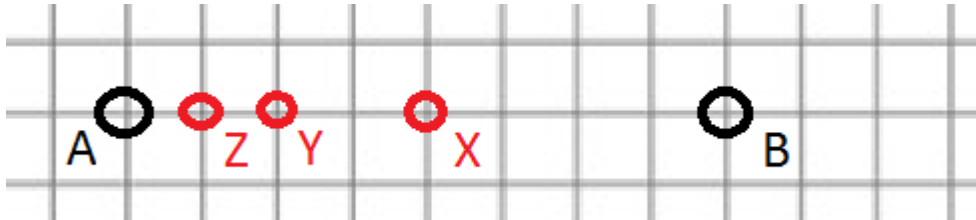
1. If you graph period squared of a pendulum on the y-axis of a graph and the pendulum's length on the x-axis, the slope of the graph equals $\frac{4\pi^2}{g}$ (and $\pi = 3.1416$, approximately). Use a slope of $4.3 \text{ s}^2/\text{m}$ and find the value of g , the acceleration due to gravity. Does it equal what you expect?
2. A vibrating tuning fork performs 2048 complete oscillations (cycles) in 4 seconds. (a) Find its frequency and period and (b) find the wavelength of the sound it makes (assuming you're in room temperature air).
3. Draw two waves that show the following (and label your waves A and B): (a) A and B have the same wavelength but A has twice the amplitude. (b) A and B have the same amplitude but A has twice the wavelength.
4. Complete the table for these waves.

λ (m)	f (Hz)	Speed (m/s)	Type of wave
1.50			Sound in air (20°C)
0.15			Sound in air (20°C)
4.0×10^{-7}		2.0×10^8	Light through glass
0.20			Sound in air (0°C)
	$* 3 \times 10^{14}$	3.0×10^8	EM in vacuum
550×10^{-9}			EM in vacuum
	2×10^{18}		EM in vacuum

5. For the * entry above, what kind of electromagnetic radiation is that? UV or Infrared?
6. In problem 4, identify the part of the electromagnetic spectrum the waves on the last two rows of the table come from.

7. Suppose you have a frequency being played that's 172 Hz and the speed of sound in air is 344 m/s. (a) What's the wavelength of this sound?

Two speakers play this sound in phase at points A and B four meters apart. Would a listener hear a **loud** or **soft** sound at (b) point X, halfway in between, (c) point Y, 1 m away from A, and (d) point Z, 0.5 m away from A?



8. Assume you stand between two loudspeakers that are excited by the same oscillator emitting an 1150 Hz sound frequency. You stand at a distance of 4 m from one of the loudspeakers. (a) What is the wavelength of the sound waves? Assume the speed of sound is 343 m/s. (b) At what *minimum* distance from you should the second loudspeaker be placed to produce destructive interference?
9. A 440 Hz tuning fork is played at the same time as a fork of unknown frequency. If a listener hears 5 beats every 10 seconds, (a) find the beat frequency, (b) find the frequency of the unknown fork if you hear it is flat ($f < 440$ Hz), and (c) find the frequency of the unknown fork if you hear it is sharp ($f > 440$ Hz).
10. Two guitars are playing together but one is tuned slightly higher than the other. If the first guitar plays an A of 110 Hz and the other plays the same A at 113 Hz, obtain the frequency heard and compute how many beats are heard in 5 seconds.
11. Four waves are described by the following functions in which all distances are measured in cm and all times in seconds.

$$y_A(x, t) = 10 \cos(3x - 4t)$$

$$y_B(x, t) = 10 \cos(5x + 4t)$$

$$y_C(x, t) = 20 \cos(-10x + 60t)$$

$$y_D(x, t) = 20 \cos(-4x - 20t)$$

- (a) Which wave has the highest frequency? (b) Which wave has the greatest speed?

12. These square-shaped wave pulses are heading toward each other in a medium whose wave speed is one grid unit per second. Using the principle of superposition, draw the resulting shape of the medium one, two, three, and four seconds later.

